

Kentucky Statewide College-Readiness Standards in Mathematics

- 1. The following are essential gateway mathematical skills that students should have in order to avoid placement into remedial (non-credit-bearing) mathematics courses and to succeed in *any* entry-level, credit-bearing mathematics course. Students who do not develop these skills in high school significantly reduce their chances for success in college and restrict their choice of college majors and career options.**

It is essential that an entering college student can do the following.

Number Sense and Numerical Operations

- A. Compute fluently and accurately with rational numbers without a calculator:
 - Add, subtract, multiply and divide integers, fractions and decimals.
 - Calculate and apply ratios, proportions, rates, and percentages to solve problems.
 - Use the correct order of operations to evaluate arithmetic expressions, including those containing parentheses.
 - Explain and apply the basic number theory concepts such as prime number, factor, divisibility, least common multiple, and greatest common divisor.
 - Multiply and divide numbers expressed in scientific notation.
- B. Recognize and apply magnitude (absolute value) and ordering of real numbers:
 - Locate the position of a number on the number line, know that its distance from the origin is its absolute value, and know that the distance between two numbers on the number line is the absolute value of their difference.
 - Determine the relative position on the number line of numbers and the relative magnitude of numbers expressed in fractional form, in decimal form, as roots, or in scientific notation.
- C. Understand that in order to solve certain problems and equations, number systems need to be extended from whole numbers to the set of all integers (positive, negative, and zero), from integers to rational numbers, from rational numbers to real numbers (rational and irrational numbers), from real numbers to complex numbers, and define and give examples of each of these types of numbers.

Algebra

- D. Perform basic operations with algebraic expressions fluently and accurately:
 - Understand the properties of integer exponents and roots and apply these properties to simplify algebraic expressions.
 - Understand the properties of rational exponents and apply these properties to simplify algebraic expressions.
 - Add, subtract, and multiply polynomials. Divide a polynomial by a low degree polynomial.

- Factor polynomials by removing the greatest common factor and factor quadratic polynomials.
 - Add, subtract, multiply, divide, and simplify rational expressions.
 - Evaluate polynomial and rational expressions and expressions containing radicals and absolute values—at specified values of their variables.
- E. Understand functions, their representations, and their properties:
- Recognize whether a relationship given in symbolic or graphical form is a function.
 - Understand functional notation and evaluate a function at a specified point in its domain.
- F. Apply basic algebraic operations to solve equations and inequalities:
- Solve linear equations and inequalities in one variable including those involving an absolute value.
 - Solve an equation involving several variables for one variable in terms of the others.
 - Solve systems of two linear equations in two variables.
 - Solve quadratic equations in one variable.
- G. Graph a variety of equations and inequalities in two variables, demonstrate understanding of the relationships between the algebraic properties of an equation and the geometric properties of its graph, and interpret a graph:
- Graph a linear equation and demonstrate that it has a constant rate of change.
 - Understand the relationship between the coefficients of a linear equation and the slope and x and y intercepts of its graph.
 - Understand the relationship between a solution of a system of two linear equations in two variables and the graphs of the corresponding lines.
 - Graph the solution set of a linear inequality.
 - Graph a quadratic function and understand the relationship between its real zeros and the x-intercepts of its graph.
 - Graph exponential functions and identify their key characteristics.
 - Read information and draw conclusions from graphs; identify properties of a graph that provide useful information about the original problem.
- H. Solve problems by converting the verbal information given into an appropriate mathematical model involving equations or systems of equations; apply appropriate mathematical techniques to analyze these mathematical models; and interpret the solution obtained in written form using appropriate units of measurement:
- Recognize and solve problems that can be modeled using a linear equation in one variable, such as time/rate/distance problems, percentage increase or decrease problems, and ratio and proportion problems.
 - Recognize and solve problems that can be modeled using a system of two equations in two variables.
 - Recognize and solve problems that can be modeled using a quadratic equation, such as the motion of an object under the force of gravity.
 - Recognize and solve problems that can be modeled using an exponential function, such as compound interest problems.

Geometry

- I. Understand the different roles played by axioms, definitions, and theorems in the logical structure of mathematics, especially in geometry:
- Identify, explain the necessity of, and give examples of definitions, axioms, and theorems.
 - State and use key basic theorems in geometry such as the Pythagorean theorem, the sum of the angles of a triangle is 180 degrees, the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.
- J. Identify and apply the definitions and properties related to lines and angles and use them to solve problems:
- Identify and apply properties of and theorems about parallel lines.
 - Identify and apply properties of and theorems about perpendicular lines.
 - Identify and apply properties of and theorems about angles.
- K. Understand the basic theorems about congruent and similar triangles and use them to solve problems.
- L. Understand the definitions and basic properties of a circle and use them to solve problems.
- M. Apply the Pythagorean theorem, its converse, and properties of special right triangles to solve problems.
- N. Use the concept of similarity of figures to solve problems.
- O. Know that geometric measurements (length, area, perimeter, and volume) depend on the choice of a unit and that measurements made on physical objects are approximations; calculate the measurements of common plane and solid geometric figures:
- Understand that numerical values are associated with measurements of physical quantities and must be assigned units of measurement or dimensions; apply such units correctly in expressions, equations, and problem solutions that involve measurements; and convert a measurement using one unit of measurement to another unit of measurement.
 - Determine the perimeter of a polygon and the circumference of a circle, the area of a rectangle, circle, triangle, and a polygon with more than four sides by decomposing it into triangles, the surface area of a prism, a pyramid, a cone, and a sphere, and the volume of a prism (for example, a rectangular box), a pyramid, a cone, and a sphere.
 - Know that the effect of a scale factor k on length, area, and volume is to multiply each by k , k^2 , and k^3 , respectively.
- P. Represent geometric objects and figures algebraically using coordinates; use algebra to solve geometric problems:
- Express the intuitive concept of the “slant” of a line in terms of the precise concept of slope, use the coordinates of two points on a line to define its slope, and use slope to express the parallelism and perpendicularity of lines.

- Describe a line by a linear equation.
- Find the distance between two points using their coordinates and the Pythagorean theorem.
- Find an equation of a circle given its center and radius.

Q. Understand basic right-triangle trigonometry and apply it to solve problems:

- Understand how similarity of right triangles allows the trigonometric function, sine, cosine, and tangent to be defined as ratios of sides, and be able to use these functions to solve problems.
- Apply the trigonometric functions sine, cosine and tangent to solve for an unknown length of a side of a right triangle, given one of the acute angles and the length of another side.

Data Interpretation, Statistics, and Probability

R. Explain and apply quantitative information:

- Organize and display data using appropriate methods (including spreadsheets) to detect patterns and departures from patterns.
- Read and interpret tables, charts, and graphs.
- Compute and explain summary statistics for distributions of data including measures of center (mean, median) and spread (range, percentiles, variance, and standard deviation).

2. The following skills are valuable but can be acquired while enrolled in a college-level, credit-bearing course. This does not, however, minimize their importance in the high school curriculum.

Number Sense and Numerical Operations

A. Understand the capabilities and the limitations of calculators and computers in solving problems:

- Use calculators appropriately and make estimations without a calculator regularly to detect potential errors.
- Use graphing calculators and computer spreadsheets.

Algebra

B. Perform basic operations on algebraic expressions fluently and accurately (additional skill):

- Derive and use the formulas for the general term and summation of finite arithmetic and geometric series and infinite geometric series with common ratio r in the interval $(-1,1)$.

C. Graph a variety of equations and inequalities in two variables, demonstrate understanding of the relationships between the algebraic properties of an equation and the geometric properties of its graph, and interpret a graph (additional skill):

- Graph ellipses and hyperbolas whose axes are parallel to the x and y axes and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics.
- D. Solve problems by converting the verbal information given into an appropriate mathematical model involving equations or systems of equations; apply appropriate mathematical techniques to analyze these mathematical models; and interpret the solution obtained in written form using appropriate units of measurement (additional skills):
- Recognize and solve problems that can be modeled using a finite geometric series, such as home mortgage problems and other compound interest problems.
 - Recognize and solve problems that can be modeled using an exponential function but whose solution requires facility with logarithms, such as exponential growth and decay problems.

Geometry

- E. Visualize solids and surfaces in three-dimensional space when given two-dimensional representations (for example, nets, multiple views) and create two-dimensional representations for the surfaces of three-dimensional objects.

Data Interpretation, Statistics and Probability

- F. Explain and apply quantitative information (additional skills):
- Compare data sets using graphs and summary statistics.
 - Create scatter plots, analyze patterns, and describe relationships in paired data.
 - Know the characteristics of the Gaussian normal distribution (bell-shaped curve).
- G. Explain and criticize alternative ways of presenting and using information:
- Evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed.
 - Identify and explain misleading uses of data.
 - Recognize when arguments based on data confuse correlation with causation.
- H. Explain the use of data and statistical thinking to draw inferences, make predictions, and justify conclusions:
- Explain the impact of sampling methods, bias, and the phrasing of questions asked during data collection and the conclusions that can be rightfully made.
 - Design simple experiments or investigations to collect data to answer questions of interest.
 - Explain the differences between randomized experiments and observational studies.
 - Construct a scatter plot of a set of paired data and if it demonstrates a linear trend, use a graphing calculator to find the regression line that best fits this data. Recognize that the correlation coefficient measures goodness of fit and explain when it is appropriate to use the regression line to make predictions.

- I. Explain and apply probability concepts and calculate simple probabilities:
- Explain how probability quantifies the likelihood that an event occurs in terms of numbers.
 - Explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome.
 - Explain how the Law of Large Numbers can be applied in simple examples.
 - Apply probability concepts such as conditional probability and independent events to calculate simple probabilities.
 - Apply probability concepts to practical situations to make informed decisions.
3. **The following mathematical skills are essential, together with the skills in category 1, for students whose intended majors require calculus and who expect to begin college taking calculus. Such majors include mathematics, physics, chemistry, computer science, engineering, biology, and business.**

Algebra

- A. Understand functions, their representations, and their properties (additional skills):
- Determine the domain of a function represented in either symbolic or graphical form.
 - Combine functions by composition, as well as by addition, subtraction, multiplication, and division.
 - Identify whether a function has an inverse and when functions are inverses of each other; explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.
 - Know that the inverse of an exponential function is a logarithmic function, prove basic properties of a logarithm using properties of its inverse, and apply those properties to solve problems.
- B. Understand the Binomial theorem and its connections to combinatorics, Pascal's Triangle, and probability.

Geometry

- C. Understand the different roles played by axioms, definitions, and theorems in the logical structure of mathematics, especially in geometry (deepened skill):
- Prove key basic theorems in geometry such as the Pythagorean theorem, the sum of the angles of a triangle is 180 degrees, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.
 - Prove basic theorems about congruent and similar triangles (deepened skill).
 - Prove basic theorems about circles (deepened skill):
- D. Use rigid motions (compositions of reflections, translations, and rotations) to determine whether two geometric figures are congruent and to create and analyze geometric designs.

- E. Represent geometric objects and figures algebraically using coordinates; use algebra to solve geometric problems (additional skill):
- Given an equation of a circle, find its center and radius (deepened skill).
- F. Know how the trigonometric functions can be extended to periodic functions on the real line, derive basic formulas involving these functions, and use these functions and formulas to solve problems.
- Know that the trigonometric functions sine and cosine, and thus all trigonometric functions, can be extended to periodic functions on the real line by defining them as functions on the unit circle, that radian measure of an angle between 0 and 360 degrees is the arc length of the unit circle subtended by that central angle, and that by similarity the arc length “s” of a circle of radius “r” subtended by a central angle of measure t radians is $s=rt$.
 - Know and use the basic identities, such as $\sin^2(x) + \cos^2(x) = 1$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$, and formulas for sine and cosine, such as addition and double angle formulas.
 - Graph sine, cosine, and tangent as well as their reciprocals cosecant, secant, and cotangent functions; identify key characteristics.
 - Know and use the law of cosines and the law of sines to find missing sides and angles of a triangle.